NOTES ON A LOGIC OF OBJECTS

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1. Introduction

Since the advent of computers and their application, in particular to artificial intelligence, it is being widely recognized that mathematical logics, such as predicate calculus, are not expressively rich enough to capture our intuitions about real world objects. Thus, researchers in AI have either abandoned predicate calculus as a basis for developing systems for reasoning about real objects or have attempted in an adhoc fashion to enhance predicate calculus by adding new primitives including concept formation, abstraction, modalities, circumscription, etc.

We think that laws of logic about the real world have a basis which is extralogical and that cannot be anything else but the real world. Here we are concerned with the *logical* laws governing real objects; this is to be distinguished from laws of physics, chemistry or other physical sciences.

These notes are an initial attempt to develop an ontological structure and propose a formalism which captures this ontology and which is governed by the same ontological structure. The basic premise on which this development is based is that corresponding to every ontological structure, there is a logical structure and linguistic structure induced by the ontology. The discussion is thus divided into three major sections, "Ontology," "Logic," and "Language."

The "logic of objects" sketched in these notes is intended as one of the cornerstones of natural logic, a novel logical formalism being developed as the basis for an approach to building system specifications and a computer language based on that approach, called Tecton [References 1,2,4,5].

Another cornerstone of natural logic is a new approach to modal logic, i.e., the use of attributes attached to propositions, called modalities. Examples of modalities are: true, false, contrary, necessary, contingent, possible, impossible, provable, inconsistent, deterministic, absurd, meaningful, etc. In representing knowledge for reasoning about systems (including real world systems), many other modalities, such as default (normal), probable, plausible, desirable, interesting, etc., turn out to be useful. Modalities are not discussed in these notes, but are occasionally used in definitions. See reference [3] for the definitions of modalities that are assumed here.

2. Ontology

Our world view commits to the philosophical principle that all our knowledge is rooted in the real world. Thus, objects in the real world (henceforth called real objects) are the most significant. Then comes what are called modes, things which do not exist by themselves but "in" objects.

Concepts, or conceptual objects, are formed to represent real objects or by transforming concepts so formed to get new concepts. Further, relationships that hold among concepts are based on real relationships among real objects.

2.1 Real Objects

Intuitively, we describe an object which existed, exists or may exist in the real world as a real object. Further, real objects get created and destroyed by natural phenomena and actions of real objects. Real relations among real objects are called connections. Like real objects, they get created and destroyed by natural phenomena and actions of real objects.

2.1.1 Parts A central ontological relation among real objects is the "is a part of" relation among objects; this relation, called the whole part relation in philosophical circles has been extensively debated. A well known axiomatization of this relation devised by the Polish logician S. Lesniewski and later by A. Tarski, is essentially based on a set theoretic interpretation of the world. In their view, for every set of objects, there exists another object which includes all elements of this set as its parts. As pointed out by Rescher, their mereology sufferes from serious shortcomings. The following example illustrates this:

In Lesniewski and Tarski's view, Carter's head, which is an object and is a part of Carter, and Reagan's heart, which is an object and a part of Reagan, form an object consisting of Carter's head and Reagan's heart, which is contrary to natural ontological intuition we possess. The problem arises because of the law of comprehension that Lesniewski and Tarski obtain in their axiomatic theory which implies that objects are constructed out of the blue by predication.

A real object can have parts which are (i) real objects, and (ii) which are connected. Formally, this is expressed by

Axiom of Realness: For every part p of x, there exists another part p' (different from p) such that p and p' are connected and the transitive closure of these (direct) connections relates every part of x to every other part x.

Further, anything which has a real object as its part is a real object itself.

Axiom of Connectedness: For every proper subset of parts of an object there is a part in the subset which is connected with some part outside of the subset.

All connections among the parts of an object constitute the form of an object.

2.1.2 Subparts Definition: A subpart p of a real object x is either (i) a part of x, or (ii) is a subpart of some part p' of x.

Axiom of Non-circularity: No real object is a subpart of itself.

Theorem: For any two real objects x and y, if x is a subpart of y, then y is not a subpart of x.

2.1.3 Integral and Essential Parts Definition: Integral parts of an object are those parts of the object needed to realize its primary purpose. Connections among integral parts constitue the integral form of the object.

Essential Parts and Essential Form:

Two intuitive constraints that we have on the definition of essential parts are (i) for certain objects, it is possible to take them apart which would result in their losing their identity and later they could be brought together which would imply their regaining their identity. This allows objects to exist, disappear and later reappear; thus there is a discontinuity in their existence. (ii) some essential parts of an object can be replaced one by one without the object losing its identity.

To define identity across time, we introduce the notion of essential parts and essential form.

Definition: An essential part of an object is an integral part such that if it is removed, the object loses its identity, hence it disappears. This is not to deny that essential parts do not change.

If there is no discontinuity in the existence of an object over a time period, the essential form defines the identity of the object because it is possible throughout this period to point to the object; otherwise if there is a gap during which an object disappeared and later it reappeared, the essential parts as well as the essential form of the object define its identity. Essential parts start playing a crucial role as they distinguish objects with identical essential form that one can obtain using equal nonidentical essential parts from the object that disappeared.

2.2 Classifications of Real Objects

2.2.1 Actual vs Possible Objects

An object that exists is called actual whereas an object that may exist or may have existed is called possible. There are two kinds of possible objects: (i) intrinsically possible object - an object whose existence is not precluded because of any contradiction being implied by its concept, and (ii) extrinsically possible object - an object that can be brought to exist by actions of some actual objects.

2.2.2 Primary Objects

Definition: Two objects x and y are disjoint if and only if they have no subpart in common.

Theorem: x is a subpart of y if and only if for every object z such that z and y are disjoint, then z and x are also disjoint.

Note that the above theorem is given as the second axiom in LT's mereology. In our world, the first axiom in LT's axiomatics does not hold because we think the requirement on objects to be part of themselves is an artificial one. In fact, the negation of the their first axiom is one of our axioms. Rescher also makes a similar criticism, but he still includes it in an axiomatization he proposes in an attempt to rectify LT's axiomatics.

Definition: A real object x is primary if and only if x is disjoint from every other real object y (different from x) such that y is not a subpart of x or x is not a subpart of y.

Intuitively, by a primary object, we mean an object x which constrains its parts, and furthermore, x is the only object constraining its parts. To what extent x constrains its parts is determined by the connections. There are a set of attributes of x which can be used to determine the attributes of x's parts using the connections.

<u>Definition</u>: A system is a real object x which need not be disjoint from any object y which is not a subpart of x nor x is a subpart of y.

<u>Theorem</u>: An object which is not primary is a system.

<u>Definition</u>: A system is strong if it is disjoint from any similar system. (The definition of similarity is discussed later.)

2.2.3 <u>Homogenity vs Heterogenity</u> <u>Definition</u>: A real object can be homogeneous or heterogeneous. In a homogeneous real object, every part is of the same species as the object itself. (The definition of species is given later.) An object that is not homogeneous is heterogenous. An example of a homogeneous object is a chunk of cheese.

2.3 <u>Real Relations among Real Objects</u>

<u>Definition</u>: Two real objects are <u>identical</u> at a given time if they have identical parts and identical forms.

If two objects have identical parts at a given time, then they also have identical forms.

<u>Theorem</u>: Every object is identical to itself.

<u>Lemma</u>: Two primary objects A and B are identical if and only if there is a part of A which is also a part of B or vice versa.

<u>Lemma</u>: If primary objects A and B have the same subpart, then A is either identical to B or subpart of B or vice versa.

Proof: by induction on subpart relation.

<u>Definition</u>: Similarity: Two objects with identical forms are called similar.

<u>Definition</u>: Equality: Two similar objects are equal if they have equal parts.

Equal nonidentical objects are distinguished from identical objects because they may have different external connections whereas for identical objects, the external connections are also identical.

2.4 <u>Modes</u>

Besides objects, there are other things which are real (not merely conceptual) but they do not exist by themselves, instead they exist in objects. For example, color, shape, weight, speed, location etc. We will call such a thing a mode (also traditionally called accident).

A mode is essential for a object if without it, the object loses its identity.

2.5 Conceptual Objects

Apart from real objects, there are conceptual objects in our ontology. They are discussed extensively in the next section on logic.

We will often use the term 'concept' for a conceptual object.

3. Logic

For any real object x, if there is a corresponding individual concept c of the object x, then for every connection of x, there is a corresponding concept which is a part of c. For every part p of x, the concept of 'having p' is a part of c. (Abstraction is usually not arbitrary. A natural way to abstract from individual concepts is via their part-form components.)

A correspondence between a real object A and its concept B can be established in two ways:

(i) extra-linguistic operation: that is B (pointing mechanism, applied to A).

(ii) existential operation: B exists (meaning that it is possible to point to the real object A that B is conceptualizing).

The connotation of a concept is the set of all subparts of the concept. (That is, it differs from the concept itself in not including the form.) Because the subpart relation is transitive, we have that if A has B in its connotation and B has C in its connotation, then A has C in its connotation.

The denotation of a concept is the set of all instances of the concept.

The universe of discourse in this terminology is a limitation by convention of what can appear in a denotation.

The connotation of a concept may include the cardinality of its denotation.

The connotation of a concept may be changed by a conceptual operation on concepts; this is to emphasize that the connotation does not change by itself whereas the denotation may change. Of course, denotation can also be changed explicitly but that is the only way connotation changes. For example, the concept of living persons whose denotation keeps changing.

Theorem: As the connotation of a concept is increased the denotation decreases (or remains the same).

3.1 Classifications of Concepts

3.1.1 Real and Logical Concepts A real concept is one that includes the concept "real object" in its connotation. The concept of a real object is a primary concept. (It does not intersect with any other concept.) The definition of a primary concept is similar to that of a primary real object.

A logical concept is one that includes the concept of concept in its connotation. It also is a primary concept.

3.1.2 Oneness, Sameness, and Existence There are three very important primary concepts which are applied to concepts: oneness, sameness, and existence.

A singular concept is a concept which includes oneness in its connotation, which means that there is at most one object in its denotation. Examples: the concepts of the highest building in Schenectady, the fastest unification algorithm, and the sorting program used by our system. (More generally, the connotation of a concept may include a concept of the size of the denotation.)

An individual concept is a concept A which includes sameness in its connotation, which means that the concept that the object that is denoted by A is always the same (identical to itself) is part of A. Examples: the concept of the father of George Washington, the concept of Sherlock Holmes, the concept of General Electric Company. Usually in the language, we designate individual concepts with proper names. Not always though, as the example of the father of George Washington shows. Concepts that are not individual concepts are never designated by proper names.

The concept of the capital of France is a singular concept but not an individual concept, since it might move from Paris.

An existential concept is a concept which includes existence in its connotation, which means that the concept of having an instance is part of the concept.

Normally, individual concepts include existence. For example, A. Conan Doyle is an individual. Sherlock Holmes is also, but is an exception in being non-existent.

Normally, existence is not an essential part of a concept.

If sameness or oneness is a part of a concept, it is an essential part.

3.1.3 Clarity, Obscurness, Distinguishability, Arbitrariness A concept is called clear (recursive) if it is possible to decide whether any given object, whether that object is in the denotation of the concept.

A concept that is not clear is called obscure.

A clear concept is called distinct if it includes in its connotation some essential properties of objects in its denotation.

A clear concept that is not distinct is called arbitrary.

3.1.3.1 Completeness, Consistency and Contradiction A distinct concept is called complete if it includes in its connotation all essential properties of objects in its denotation.

A distinct concept that is not complete is called incomplete.

A concept is called contradictory if there is a property in its connotation such that the negation of the property is also in its connotation.

A concept that is not contradictory is called non-contradictory.

Note that the properties contradictory and non-contradictory are proof-theoretic.

A concept is called consistent if every object in its denotation satisfies every property in its connotation.

A concept that is not consistent is called inconsistent.

Note that the properties consistent and inconsistent are model-theoretic. A consistent concept may become inconsistent independent of any conceptual operation because, as noted above, the denotation can also change impliciitly. This is in contrast with connotation which can only change because of a conceptual operation, so a non-contradictory concept can never become contradictory without an explicit conceptual operation.

A concept is strongly complete if no further property can be added to the concept without making it contradictory.

3.2 Relations Among Concepts

Concepts can be related in two different ways based on relations of their connotations or denotations. For a relation R on concepts, a concept A is c-R related to a concept B if connotation of A is R-related to connotation of B, and similarly, A is d-R related to B if denotation of A is R-related to denotation of B.

Axiom Schema: Normally, for any relation R, c-R implies d-R.

Different kinds of R:

subsetting: subset

intersecting but nonsubsetting:

disjointness:

identical: obviously equal

nonidentical:

equal: weaker sense

unequal:

similar: treats parts as variables but keep the connections among parts invariant

dissimilar:

contradictory: two concepts are contradictory if one includes in its connotation A whereas the other includes ~A, but they are equal everywhere also.

denotationally contradictory: the denotation of the proximate genus is partitioned using the two concepts.

contrary: dual : greatest vs smallest

general concept: concept whose denotation may include more than one object.

collective concept: a constructor which operates only on general concepts to give a singular concept, for example, library which is obtained from books.

substantial concept: conceptualization of matter that is homogeneous and taking out a portion of it does not change its substance, example water, gold, etc.

Concepts A and B are <u>connotationally equivalent</u> if and only if for any concept C, A is C if and only if B is C.

Informally, connotational equivalence captures the intuitive notion that two concepts are the same when they are subjected to any property expressible in the language. Such an equivalence can be verified purely by deduction.

Examples of pairs of concepts that are connotationally equivalent:

"set accepted by a finite automaton" and "regular set"

"mother-in-law" and "wife of husband's father or wife of wife's father"

Theorem: For any concepts A and B, A and B are connotationally equivalent if and only if A is B and B is A.

Proof: Assume A and B are connotationally equivalent. Then, in the definition of connotational equivalence take C to be A:

A is A if and only if B is A.

Since A is A is axiomatic, we have B is A. Similarly, A is B.

In the other direction, suppose A is B and B is A. Let C be a concept such that A is C. Then B is C also, by transitivity of "is." Thus A is connotationally equivalent to B. Q.E.D.

Concepts A and B are denotationally equivalent if and only if for any instance X, X is A if and only if X is B.

Denotational equivalence captures the intuition that two concepts have the same denotations (the same instances). Denotational equivalence can be verified by observations.

Denotational equivalence can change without having to redefine the concepts but because the properties (attributes) of concepts and instances change. This is in contrast to the connotational equivalence which can only change if the related concepts are redefined, or in other words, by a conceptual operation. Examples: the concept of the sun was once that of a celestial body that revolves around the earth, then it was changed to a celestial body around which the earth revolves. The connotation changed, but the denotation stayed the same.

Connotational equivalence implies denotational equivalence, but not vice versa. To prove the first part of this statement, assume that A is connotationally equivalent to B. By the theorem of the previous subsection, A is B and B is A. Let X be an entity such that X is A. By transitivity of "is," X is B. By a symmetric argument, if X is B then X is A. Thus A and B are denotationally equivalent.

3.3 Concept formation Operations

There are five classes of predication of concepts: genus, species, difference, property, accident.

1. Species of an object - all essential parts in the connotation of an individual object.

2. Genus of an object - part of connotation of species which it shares with some other species.

Ordering on essential parts in the connotation of an object gives this tree of genera; different orderings may give different trees.

Proximate genus - genus nearest to the species, i.e., one obtained by not considering only the least essential part in the connotation of the object.

Remote genus - genus farthest to the species in the tree, i.e., one obtained by considering only the most essential part.

3. Difference - part of connotation of species which distinguishes it from any other species in the proximate genus.

4. Property of an object - some attribute necessarily shared by all denotations of its species.

5. accident of an object - attribute in the connotation which is neither essential nor a property.

Classifications of accidents:

1) mutable vs immutable
2) sharable vs nonsharable

4. Language

4.1 Terms

A term denotes a concept. Terms can be classified in many ways based on different classifications of concepts denoted by them. Further, if a term denotes a concept of a particular kind, then the term is called of that kind. For example, a term denoting a singular concept is called a singular term.

Terms can be one of the following:

1) atomic term;
2) compound term:
   a) terms connected with a conjunct;
   b) qualified term;
   c) quantified term.

4.1.1 Atomic Terms A term is atomic if and only if no part of it is a term (i.e., no part denotes a concept). Atomic terms can be classified based on their denotations, so we will use the classification discussed in the previous section whenever the need arises.

Among atomic terms, we distinguish atomic terms which are proper names and which denote individual concepts. However, there are individual concepts for which there may not be any proper name. It seems convenient to start a proper name with capital letters.

4.1.2 Compound Terms A compound term is built from atomic terms using conjuncts, qualifiers and quantifiers. The syntactic structure of a compound term correpsonds to the conceptual operations on concepts to give other concepts.

4.1.2.1 Qualifiers A qualifier corresponds to the refinement operation; given a term corresponding to a concept, a qualified term denotes the refined concept. They are added to terms with the help of conjunct "such that" and are propositions in which pronouns are bound over the qualified term. For example, "programs, such that any verifier cannot verify them".

Among qualified terms, it is possible to distinguish between those obtained after qualifing absolute terms (terms denoting concepts that are not constructors) and those obtained by application of a term denoting a constructor (relative term) on another term denoting a concept.

4.1.2.2 Quantifiers A quantifier corresponds to the conceptual operation which when applied on a concept results in a collective concept. A quantified term has two parts: a quantifier, which determines the type of quantification, followed by unquantified term.

Quantified terms cannot be qualified or quantified. (In general, singular terms cannot be qualfied and quantified; a quantified term is a singular term.)

The type of quantification gives information about the cardinality of the collective concept that the qunatified term denotes. We now discuss different kinds of qunatifications.

4.1.2.2.1 _Universal quantifiers_ Universal quantified terms are introduced by quantifiers "all", "every", "any". There is a difference between "all" and "every" and "any." "All" gives a set, while "every" and "any" give any element from the set. For example, "all members of CSB ate 25 hamburgers" is quite different from "every member of CSB ate 25 hamburgers."

4.1.2.2.2 _Existential quantifiers_ Existential quantified terms are introduced by quantifiers "some", "a", and "an". (Note that indefinite article is not equivalent to "any").

4.1.2.2.2.1 _Singular existential quantifiers_ Singular existential quantified terms are introduced by quantifiers "a", "an", and "one". They denote a singular concept. For example, "a man", "one big computer".

4.1.2.2.2.2 _Plural existential quantifiers_ Plural existential quantified terms are introduced by quantifiers "some", and "some of". They specify one noneempty subset of objects of a given type. For example, "some natural numbers".

4.1.2.2.3 _Numerical quantifiers_ Numerical quantifiers are a refinment of existential quantifiers, namely, for any numerical quantifier A and any term t A(t)=>some(t). Numerical quantifiers specify a non-empty subset of certain cardinality of objects of given type.

4.1.2.2.3.1 _Exact numerical quantifiers_ Exact numerical quantifiers specify cardinality. They are introduced by a cardinal or by a construct "as many <type description> as <set description>. If in the second case a set is empty, then the construct is equivalent to negative quantified term. For example, "3 men", "as many hamburgers as people in CSB".

4.1.2.2.3.2 _Relational numerical quantifiers_ Relational numerical quantified terms are introduced by syntactic constructs "<comparator> <type description> then <set description>" or "<comparator> then <cardinal numeral> <type description>", where comparators are "more", "less", "more or equal" and so on. For example, "less then 5 men", "less hamburgers then people in CSB".

4.1.2.3 _Conjunction of Terms_ A conjunct "A and B" of terms A and B denotes the union of concepts denoted by A and B.

4.1.2.3.1 Disjuncts Noun disjunct "A or B" is nondeterministic construct which gives as its result one of three choices: A, B, A and B.

4.1.2.4 Negation A negation of a term denotes the concept which is contradictory to the concept denoted by the term.

4.1.2.5 Parentheses in terms Parenthesis can be used to disambiguate application of quantifiers and qualifiers to composite terms. "Stupid man or woman" means "(stupid man) or woman" or "stupid (man or woman)".

4.1.3 Supposition of Terms There are three ways a term can be used; these different ways, which are traditionally called suppositions, can be disambiguated, whenever the need arises, by using different kinds of quoting mechanisms. For example, in the proposition

computer scientists are smart,

computer scientists is a real supposition, whereas in

'computer scientist' is not a species,

computer scientist is a logical supposition, and in

"computer scientist" is not an atomic term,

computer scientist is a material supposition.

The convention we adopt is that if a term does not have any quotes around it, then it is usually meant to be in a real supposition, whereas if a term has single quote marks (') around it, it is then meant to be in a logical supposition, and if a term has double quote marks (") around it, it is in a material supposition. The ability to talk about different suppositions explicitly allows us to extend syntax and semantics of the language.

4.1.4 Unequivocal and equivocal terms Terms can be classified based on the number of concepts they denote. A term that denotes one concept is called univocal, whereas a term that denotes more than one concept is called equivocal.

4.2 Propositions

There are four different types of propositions: categorical, modal, lexical and compound. Compound propositions are formed by combining propositions using propositional conjuncts. Categorical propositions describe relations between different objects. Modal propositions describe a logical status of propositions. Lexical propositions assign meaning to sentences and other linguistic objects and are used for definitions. Each of these proposition types are discussed in more detail below.

4.2.1 Categorical propositions A categorical proposition has two parts: subject and predicate, each of which is a term. The form of a categorical proposition is called copula, which is not a term.

Two propositions are similar if they have the same copula.

Corresponding to every relation R among concepts, there are two copulas is-R and is-not-R which are used to construct categorical propositions expressing the relation between two concepts. A proposition using the copula is-R is called R-affirmative and a proposition using is-R-not is called R-negative. Whenever, there isn't any need to refer to R, we would just refer to propositions as being affirmative or negative. Whether a proposition is affirmative or negative is callled its mode.

An affirmative proposition "A is B" means that every instance in the denotation of A is in the denotation of B and every attribute (i.e., part of the connotation) of B is an attribute of every instance of A.

A negative proposition "A is not B", in contrast, means that there is an instance (in the denotation) of A which is not in an instance of B and every instance of A has "really" an attribute which is not an attribute of B which may or may not be deducible because instances of A as well as the connotation of B may not be completely known.

The above categorical propositions have been interpreted a posteriori, which will be the default. To express a priori propositions, we explicitly introduce words "a priori" to avoid ambiguity. A proposition "A is a priori B" means that the connotation of A implies the connotation of B.

4.2.2 Modal propositions Every proposition has attributes called modalities. A proposition which describes a modality of some other proposition is called a modal proposition. Examples of modalities are: true, false, contrary, necessary, contingent, possible, impossible, deterministic, absurd, and meaningful. A proposition may have more than one modality. For example, if "X" is true then "X" is possible.

The form of a modal proposition is ""P" is M", where M is a modality and P is a proposition, or simply "M, P" where M is modal adverb.

4.2.3 Compound propositions There are three kinds of compound propositions: conjunctive, disjunctive, and conditional.

4.2.3.1 Conjunctive propositions A conjunctive proposition is a list of two or more propositions separated with conjunct "and", ",", or some other conjunctive conjunct.

A conjunctive proposition is true if and only if all its parts are true. A conjunctive proposition is necessary if and only if all its parts are necessary. A conjunctive proposition is possible if and only if all its parts are possible. A conjunctive proposition is impossible if one of its parts is impossible or the negation of one part is derivable from other parts; a part of a conjunctive proposition is derivable from that conjunctive proposition.

4.2.3.2 Disjunctive propositions A disjunctive proposition is a list of two or more propositions separated with conjunct "or" or some other disjunctive conjunct. A disjunctive proposition is true if and only if one of its parts is true. A disjunctive proposition is possible if and only if one part of it is possible. A disjunctive proposition is impossible if and only if all its parts are impossible. A disjunctive proposition is necessary if and only if one of its parts is necessary or a part of it is derivable from a negation of some other part. A disjuncive proposition is derivable from any of its parts.

4.2.4 Conditional propositions A conditional proposition is a pair of propositions separated with conditional conjunct "if", "only if" or "if and only if". The consequent of a conditional proposition is defined to be its first part of the proposition in the case of the conjunct "if", the second part in the case of the conjunct "only if", and both parts of a proposition with conjunct "if and only if". The antecedent is defined to be the second part of the proposition in the case of the conjunct "if", the first part of a conditional proposition with conjunct "only if", and both parts of a conditional proposition with conjunct "if and only if". A conditional proposition is true if and only if each consequent is derivable from each antecedent.

Implication is a particular case of the part construct for logical objects of type proposition. That allows us to derive the semantics of implication. For example modus ponens becomes a particular case of more general rule for objects:

B exists if B is part of A and A exists.

The Rule of Substitution is derivable from: A is part of C if A is part of B and B is part of C.

5. Sentences

Aside from propositions, sentences of the language include imperative statements, which describe a computation or an action, and questions, which are a special kind of imperative statement which order (or request) an action. Questions have different syntax from imperative statements, but can be represented as imperative statements (for example, "what is 2 + 2?" means the same as "give the value of 2 + 2").

6. References

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